Continuity and Derivability Issues in Modeling the Energy Release Shaping Window in Stochastic Simulation of Ground Motions Generated by Vrancea Intermediate-Depth Seismic Source

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***Abstract*—When applying the stochastic method in time domain ground motions simulations, a shaping function is required for modeling the energy release during an earthquake. It was noticed that Vrancea (Romania) intermediate-depth source produces strong pulse-type accelerations that contain up to 50-70% of the energy in a first phase that lasts only few seconds (under 5 seconds). After that the motion enters in a second phase that releases the rest of the energy slowly (in 30-50 seconds). For modeling the described behavior, a multi-interval function for modeling the energy release during an earthquake was required. In this paper the problem of continuity and derivability of such window-function is analyzed and discussed.**

***Index Terms*—stochastic simulation, Vrancea earthquakes, energy release, shaping window, continuity and derivability**

I. Introduction

The stochastic simulations of ground motions produced by earthquakes can be obtained using two methods: one in time domain and the other by estimating the peak motions employing random vibration theory. In time-domain simulations a white noise is generated, windowed with a function that shapes the release of energy in time, multiplied by the ground motion's spectrum (that carry the source, path and site-conditions parameters) after being normalized in frequency domain, and transformed back in time-domain obtaining accelerograms, velocigrams or seismograms, as needed [1]. In simulations that use the random vibration theory the seismic energy described by the ground motion spectrum is distributed in accordance with the number of zero crossing, the number of extrema (consistent with their frequencies and significant durations) and the ratio of peak motion to root mean square motion in order to obtain peak values [1]. The current paper is concerned with the time domain stochastic simulations made using SMSIM set of programs [2] for ground motions generated by Vrancea (Romania) intermediate-depth seismic source. Researches in the stochastic simulations topic with respect to Vrancea (Romania) intermediate-depth earthquakes can be found in the papers of Benetatos and Kiratzi [3], Oth et al. [4], Pavel [5, 6], Pavel and Vacareanu [7, 8], Pavel et al. [9] and Coțovanu [10, 11], but the problem of the window that models the energy release was not discussed yet.

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When analyzing 371 horizontal components of ground motions recorded at different stations during March 4th 1977, August 30th 1986, May 30th and May 31st 1990, October 27th 2004, Vrancea intermediate-depth earthquakes with moment magnitude at least 6, a pattern was noticed in almost half of them (169 recordings) with respect to the cumulative normalized energies. As one may see in Figure 1, the energy is released firstly in an abrupt manner during a short period of time (a strong motion segment of a pulse-like form) and then in a slow manner during an up to ten times longer period of time.

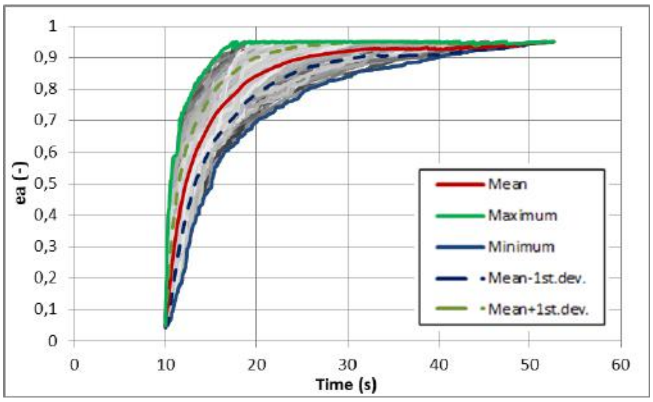

Figure 1. The recordings (shades of grey) and the statistical descriptors (mean values, mean minus/plus one standard deviation, maximum and minimum values) for the recordings where the evolution of the cumulative energies in time (analyzed fraction being 0.05-0.95) has a first abrupt release of energy and then a slowly released energy for 169 recordings from March 4th 1977, August 30th 1986, May 30th and May 31st 1990, October 27th 2004 earthquakes

In SMSIM set of programs the shape of the amplitude variation of the generated white noise in time domain (that leads the energy distribution after being loaded in frequency domain with the ground motion's spectrum) is controlled through a box or an exponential window. The implemented exponential window is derived from the paper of Saragoni and Hart [3] who determined for the expected mean square acceleration the following form:

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$$E\left[W_a(t)\right]=\int_0^t \beta\tau^{\gamma}e^{-\alpha\tau}d\tau=\beta P(\gamma+1,\alpha t)\frac{\Gamma(\gamma+1)}{\alpha^{\gamma+1}}, \quad (1)$$

where

$$P(a,x)=\frac{1}{\Gamma(a)}\int_0^x t^{a-1}e^{-t}dt \quad (2)$$

$P(a,x)$ – incomplete gamma function; $\Gamma(a)$ – gamma function.

They demonstrated that the shape of the window-function for modeling the amplitude variation over time is:

$$\psi(t)=\sqrt{\beta}t^{0.5\gamma}e^{-0.5\alpha t} \text{ (form used in SMSIM)} \quad (3)$$

where: β – intensity parameter; α, γ – shape characterization parameters; $a(t)$ – time-history ground motion acceleration.

Mathematically, the expected energy function is not able to change its slope as dramatically as it is required to describe the Vrancea-earthquakes specific release of energy pattern. For defining the ground motion shape, other functions can be found proposed by Shinozuka and Sato [4], Preumont [5] and Orabi et al. [6] and others, but none of them can describe the above mentioned pattern. For Vrancea specificity energy release a multi-interval shape function is needed. For this purpose, the problem of continuity and derivability is further addressed.

II. Continuity and Derivability Issues of the Window-Function with Two Intervals

As the shape of the function depends on the expected mean square acceleration values, the proposed window-function is based on the form from Saragoni and Hart [3] with two sets of parameters for each interval as follows:

$$\psi(t)=\begin{cases}\sqrt{\beta_1}t^{0.5\gamma_1}e^{-0.5\alpha_1 t} & t\in(0,t_0)\\ \sqrt{\beta_2}t^{0.5\gamma_2}e^{-0.5\alpha_2 t} & t\in\left[t_0,t_{final}\right)\end{cases} \quad (4)$$

$$E\left[W_a(t)\right]=\begin{cases}\beta_1 P(\gamma_1+1,\alpha_1 t)\frac{\Gamma(\gamma_1+1)}{\alpha_1^{\gamma_1+1}} & t\in(0,t_0)\\ \beta_2 P(\gamma_2+1,\alpha_2 t)\frac{\Gamma(\gamma_2+1)}{\alpha_2^{\gamma_2+1}} & t\in\left[t_0,t_{final}\right)\end{cases} \quad (5)$$

Ideally the proposed window should meet together the following conditions:

a) The expected energy function must be continuous in t_0:

$$\lim_{t\to t_0} E\left[W_a(t)\right]=E\left[W_a(t_0)\right] \quad (6)$$

b) The expected energy function must be derivable in t_0:

$$\lim_{t\to t_0} E'\left[W_a(t)\right]=E'\left[W_a(t_0)\right] \quad (7)$$

so:

$$\lim_{t\to t_0} \psi^2(t)=\psi^2(t_0) \quad (8)$$

c) If the window follows the form presented by Boore [1] the window has a local maximum in the first interval, so the derivative of the expected energy function has a local maximum (the second derivative of the expected energy function is 0 at a point t_x in the first definition range):

$\exists t_x\in(0,t_0)$, where $E''\left[W_a(t_x)\right]=0$

$$E''\left[W_a(t_x)\right]=-\beta t_x^{\gamma-1}(\alpha t_x-\gamma)e^{-\alpha t_x} \quad (9)$$

Because t_0 will be defined as a variable parameter depending on the database used to determine the parameters, for the matter of studying the problem of continuity and derivability, $t_0\in(8,10)$ is considered arbitrary.

The window parameters were determined for the mean cumulative energy of the recordings that presented the pattern described in Section I. MATLAB's curve-fitting application was used and the window parameters from Table I resulted for the two intervals, 0-10 s and 8 s-end. The correlation squared values (R-sq) are greater than 99.00% indicating a very good fit. The interpolation intervals intersect on the 8-10 s portion because a smooth transition between the two functions is aimed.

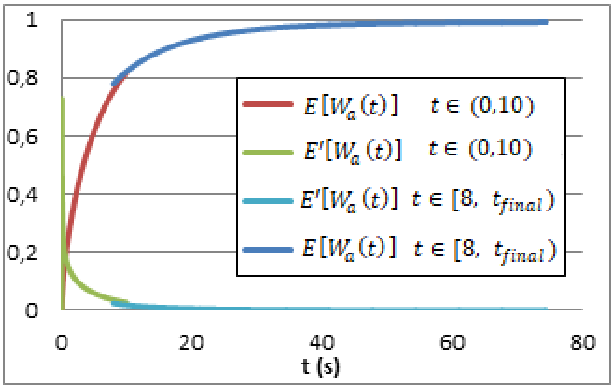

Figure 2. The expected mean square acceleration function defined on two intervals $E\left[W_a(t)\right]$ and its derivative

TABLE I. Window Parameters Determined for the Mean Cumulative Energy of Recordings that Presented the Vrancea Specific Release of Energy Pattern for the Two-Interval Function

	1st $t\in(0,10)$		2nd $t\in(8,t_{final})$
α1	0.1154	α2	0.07112
β1	0.174	β2	0.1554
γ1	–0.3065	γ2	–0.617

As one may observe from Figure 2 and from verifying the above conditions, the function thus defined is neither derivable nor continuous at any point from $t\in(8,10)$. Even if the parameters could be slightly changed (the statistical parameters of the interpolation remaining in the limits of a good fit) aiming to find a point t_0 of continuity, the function cannot be derivable at that point because by defining the shaping window on two intervals the purpose was to dramatically change the slope of the expected energy function.

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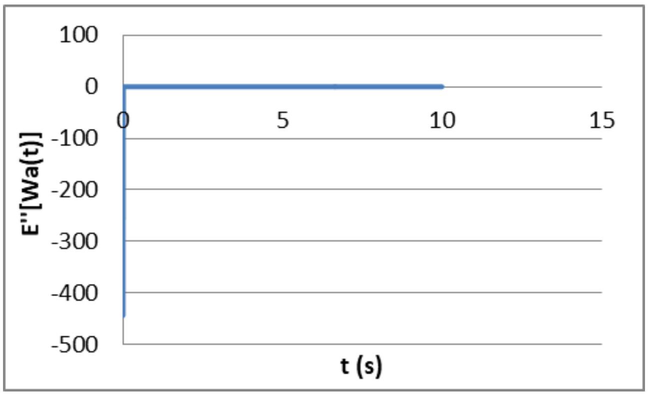

Figure 3. The second order derivative of the first interval's expected energy function $E\left[W_a(t)\right]$

As for the third condition, the second order derivative of the first interval of the expected energy function $E''\left[W_a(t)\right]$ strives asymptotically to $-\infty$ when $t \to 0$ and to 0 when $t \to \infty$ (Figure 3), so there is no maximum point in the first interval function. This means that the first point of the window depend on the time step chosen for the simulation and the window will need scaling in order to achieve a maximum equal to unity.

III. The Problem of Continuity and Derivability of the Window-Function with Three Intervals

Aiming to find a continuous and derivable window a three-interval function was defined using a polynomial linking function. For the first and third intervals the previous form of energy definition was considered, and for the second interval a 4th degree polynomial function was used as follows:

$$\psi(t) = \begin{cases} \sqrt{\beta_1 t^{0.5\gamma_1} e^{-0.5\alpha_1 t}}\, t \in (0, t_0) \\ \sqrt{3p_1 t^2 + 2p_2 t + p_3}\, t \in [t_0, t_1) \\ \sqrt{\beta_2 t^{0.5\gamma_2} e^{-0.5\alpha_2 t}}\, t \in [t_1, t_{final}) \end{cases} \tag{10}$$

$$E\left[W_a(t)\right] = \begin{cases} \beta_1 P(\gamma_1+1, \alpha_1 t) \dfrac{\Gamma(\gamma_1+1)}{\alpha_1^{\gamma_1+1}}\, t \in (0, t_0) \\ p_1 t^3 + p_2 t^2 + p_3 t + p_4\, t \in [t_0, t_1) \\ \beta_2 P(\gamma_2+1, \alpha_2 t) \dfrac{\Gamma(\gamma_2+1)}{\alpha_2^{\gamma_2+1}}\, t \in [t_1, t_{final}) \end{cases} \tag{11}$$

The proposed shaping window should meet together the following conditions:

a) The expected energy function must be continuous in t_0 and t_1:

$$\lim_{t \to t_0} E\left[W_a(t)\right] = E\left[W_a(t_0)\right] \tag{12}$$

$$\lim_{t \to t_1} E\left[W_a(t)\right] = E\left[W_a(t_1)\right] \tag{13}$$

b) The expected energy function must be derivable in t_0 and t_1:

$$\lim_{t \to t_0} E'\left[W_a(t)\right] = E'\left[W_a(t_0)\right] \tag{14}$$

$$\lim_{t \to t_1} E'\left[W_a(t)\right] = E'\left[W_a(t_1)\right] \tag{15}$$

so:

$$\lim_{t \to t_0} \psi^2(t) = \psi^2(t_0) \tag{16}$$

$$\lim_{t \to t_1} \psi^2(t) = \psi^2(t_1) \tag{17}$$

For the first and third branches the parameters were determined as described in Chapter II using Matlab for the intervals 0-8 s and 12 s-end. Those from the polynomial branch (8-12 s) resulted from the system of equations made with the continuity and the derivability conditions. In Table II the values of the parameters are given in the order of their determination (further on, the notation of the branches is illustrated in this order).

TABLE II. WINDOW PARAMETERS DETERMINED FOR THE MEAN CUMULATIVE ENERGY OF RECORDINGS THAT PRESENTED THE VRANCEA SPECIFIC RELEASE OF ENERGY PATTERN FOR THE THREE-INTERVAL FUNCTION

	1st $t \in (0,8)$		2nd $t \in (12, t_{final})$		3rd $t \in (8,12)$
α1	0.117	α_2	0.07112	p_1	0.0000736
β1	0.1746	β_2	0.1554	p_2	–0.004976
γ1	–0.3037	γ_2	–0.617	p_3	0.101878
				p_4	0.220196

Defining the function as mentioned above, the imposed conditions are met (Figure 4). But when normalized according to the method implemented in SMSIM [1], the function thus defined loses its continuity (Figure 5). However, by scaling the function to reach a unit peak the continuity is preserved (Figure 6).

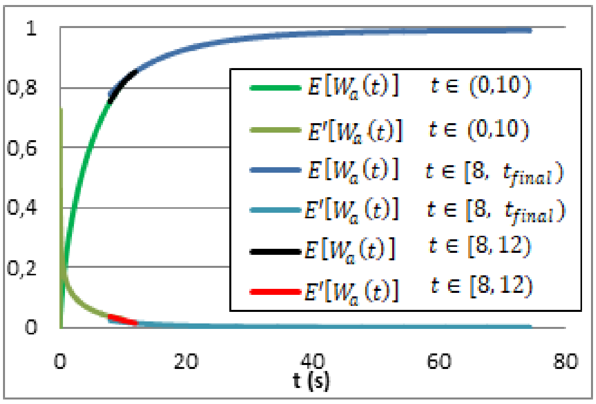

Figure 4. The expected mean square acceleration function defined on three intervals $E\left[W_a(t)\right]$ and its derivative

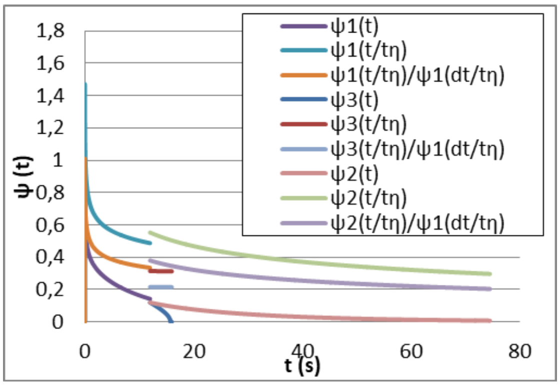

Figure 5. Comparison between the three-interval window-function $\psi(t)$, the function in normalized time $\psi(t/t_n)$ and the function in normalized time scaled to $1\,\psi(t/t_\eta)/\psi 1(dt/t_\eta)$. Number indices refer to the function branches and correspond to the order of parameter determination (1 – first interval 0-8 s, 2 – third interval 12-end, 3 – second interval 8-12 s)

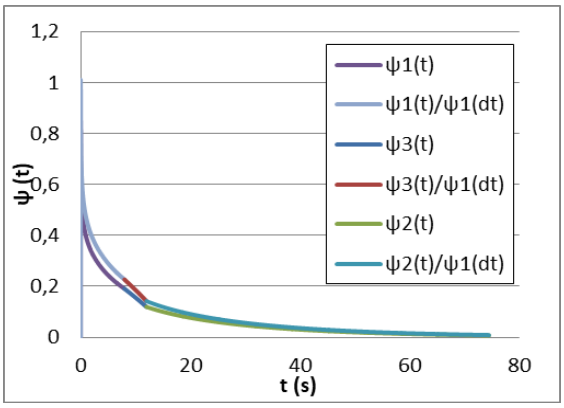

Figure 6. Comparison between the three-interval window-function $\psi(t)$ and the function scaled to $1\psi(t)/\psi 1(dt)$. Number indices refer to the function branches and correspond to the order of parameter determination (1 - first interval 0-8 s, 2 – third interval 12-end, 3 – second interval 8-12 s)

Because t_0 and t_1 are variable parameters, to verify the function's continuity and derivability for another time intervals, the values $t_0 = 12\text{ s}$ and $t_1 = 16\text{ s}$ were chosen. As one may observe in Figure 7, both the continuity and the derivability of the function were lost; even more for a time greater than 15.75 s the window-function becomes negative.

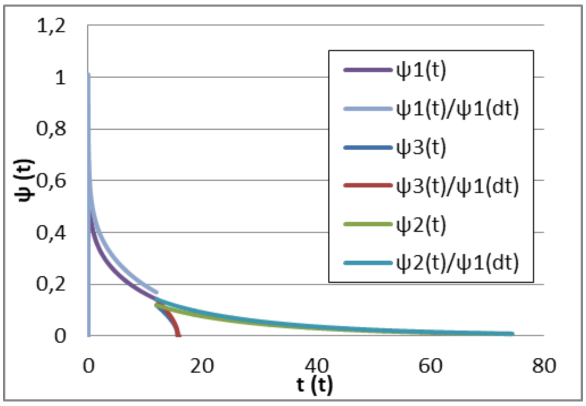

Figure 7. Comparison between the three-interval window-function $\psi(t)$ and the function scaled to $1\psi(t)/\psi 1(dt)$ for the defining intervals 0-12 s, 12-16 s and 16-end. Number indices refer to the function branches and correspond to the order of parameter determination (1 - first interval 0-12 s, 2 – third interval 16-end, 3- second interval 12-16 s)

IV. CONCLUSIONS

As shown, a multi-interval shaping window that describes the specificity of Vrancea intermediate-depth seismic source energy release can be defined, but its continuity and derivability cannot be achieved, the implications being further discussed.

The stochastic simulation in time domain in SMSIM is performed with discrete variables. Modeling the generated white noise with a shaping window that is not continuous assumes that there will be a point between t_0 and $t_0 + dt$, where the function will not coincide at limit. Because the multiplication of the white noise with the window is performed in a discrete domain, the noise has no values in the discontinuity interval; therefore the simulation will not be influenced.

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