

# Digital Terrain Modeling Using SPOT Imagery

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**Abstract**—Platforms with high-resolution recording sensors are an excellent source for an efficient, economical and accurate generation of the terrain numerical model. The most difficult problems associated with generating the numerical model of the terrain are the correlation and the establishment of the mathematical model that describes the relationship between the scene coordinates and the object coordinates. Both rigorous and approximate models can be used to describe this relationship. Rigorous modeling requires accurate knowledge of image geometry; inner and outer orientation parameters are explicitly involved. Determining these features requires control information that can be in the form of control points, and / or navigation units. However, determining these parameters can be hindered by the lack of sufficient control information, poor image geometry and/or by the provider's secrecy of these parameters. That is why there has been and still is an increased interest within the photogrammetry community for adopting approximate models; their feature being that such models do not imply any exact knowledge of the image geometry or of the internal and external characteristics of the sensor. Parallel projection is one of the approximate models that has gained popularity due to the simplicity and precision of the representation of the projection perspective characteristic of the scenes taken with sensors with a narrow angle of view that move with constant speed and orientation.

**Index Terms**—photogrammetry, epipolar geometry, satellite imagery, digital terrain model.

## I. INTRODUCTION

The main purpose of reassembling images in epipolar geometry is to obtain normalized images where the corresponding points are on the same line (or column). A first advantage is that of reducing the search space and the computation time, and therefore the ambiguity of the image correlation is reduced. This is important for a wide variety of applications, such as: automatic correlation, automatic relative orientation, automatic aerotriangulation, automatic numerical model generation, orthophoto generation, stereoscopic vision.

The sensors used on the satellite platforms for high resolution imaging are implemented as a linear string, the continuous coverage of the surface being obtained by multiple exposures during the movement of the platform on the trajectory.

For systems moving with constant speed and attitude, the geometry of the image can be described as a perspective projection in the direction of the linear string and as a parallel projection along the direction of flight (trajectory) [1] and [6].

Moreover, for systems with a narrow field of view (AFOV), such as high-resolution satellite sensors (e.g. AFOV for IKONOS is less than one degree), parallel

projection is the most protected model to describe the relationship between the scene and object coordinates [3], [4] and [5].

## II. MATHEMATICAL MODEL

### A. Transforming perspective projection into parallel projection

Perspective geometry associated with recording systems with a narrow (small) viewing angle is very close to parallel projection.

However, the recordings taken with such sensors must be modified to have geometry close to that of the parallel projection.

Transforming perspective projection into parallel projection involves transforming the recording coordinates along the scan lines, taking into account the scale variation in the scan direction.

In order to eliminate the scale variation along the scan direction and to obtain a uniform scale specific to the parallel projection, the value of the longitudinal inclination angle of the scanner (roll angle)  $\psi$  must be determined.

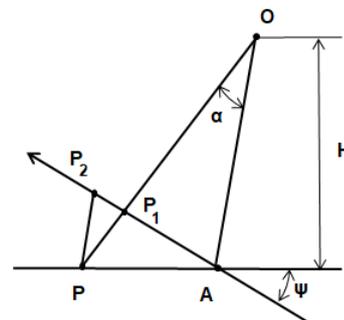


Figure 1. Perspective projection

The pickup systems perform stereo coverage in three ways, namely: by tilting the rear along the flight direction (similar to IKONOS), by tilting on both sides to the flight direction (similar to SPOT) or by using three rows front, rear, rear scan (similar to MOMS and ADS40).

The following figure shows the modification of the scale along the scan direction for the three types of modes.

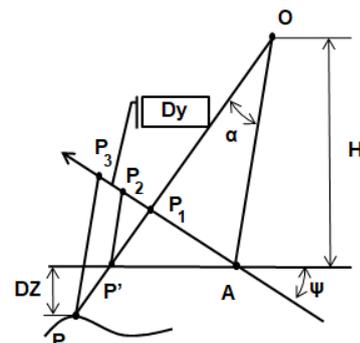


Figure 2. Perspective to parallel projection

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B. Parallel projection

The parallel projection model involves eight parameters: two direction parameters -  $L, M$ , three rotation angles -  $\omega, \phi, \kappa$ , two translations -  $\Delta x, \Delta y$  and a scale factor -  $s$  [2] and [4].

The nonlinear framing of the model can be re-parameterized to obtain a linear shape such as:

$$\begin{aligned} x &= A_1X + A_2Y + A_3Z + A_4 \\ y &= A_5X + A_6Y + A_7Z + A_8 \end{aligned} \quad (1)$$

The geometry of scenes recorded with a sensor moving along the trajectory with constant speed and orientation can be described by a parallel projection along the flight direction and by a perceptual projection along the scan direction.

Perspective projection along the scanner direction can be approximated with a parallel projection for narrow-angle systems.

In order to transform the perspective projection into a parallel projection along the scan direction into the equation of  $y$  in (1), an additional term ( $\psi$ ) is introduced; (2)

$$y = \frac{A_5X + A_6Y + A_7Z + A_8}{1 + \frac{\tan(\psi)}{c}(A_5X + A_6Y + A_7Z + A_8)} \quad (2)$$

Parameters ( $A_1 \sim A_8, \psi$ ) can be estimated for a scene if at least five control ground points are available.

One of the working hypotheses used in establishing the model of transforming the perspective projection into parallel projection was that of flatness of the hypothesis field.

However, this hypothesis, does not generally verify why the error introduced by this hypothesis was quantified and the mathematical model of the transformation was modified to include information on the relief trend; this information is extracted from the support points.

Starting from these observations, the Photo-AddValue extension was extended to allow the use in the model of transforming perspective projection into parallel projection and relief information, information obtained through the support points.

The new mathematical model for transforming perspective projection, characteristic images along the scan line into parallel projection, can be expressed as follows:

$$y = \frac{A_5X + A_6Y + A_7Z + A_8 + s\Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)}}{1 + \frac{\tan(\psi)}{c} \left( A_5X + A_6Y + A_7Z + A_8 + s\Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)} \right)} \quad (3)$$

III. GENERATION OF NORMALIZED SCENES

The simplicity of the parallel projection model allows easy generation of the normalized images within the processing process through the epipolar geometry.

The normalization process allows to reduce the search space of the conjugate point from two dimensions to a single dimension; the search is done along the epipolar lines that correspond to the same lines in the normalized scenes.

The process of processing the satellite recordings in epipolar geometry can be described by the following steps [1] and [7]:

- parallel projection parameters ( $A_1 \sim A_8, \psi$ ) are determined for the left scene and the right scene, using control field points;
- parameters  $L, M, \omega, \phi, \kappa, \Delta x, \Delta y$  are derived from the previously determined parameters ( $A_1 \sim A_8$ );
- parallel projection parameters corresponding to the left or right scene are used to determine a new set of normalized parameters. The new parameters ensure the elimination of the transverse parallax between the conjugate points, and the longitudinal parallax is linearly proportional to the altitude of the corresponding points in the object space;
- original and normalized parallel projection parameters are used to transform the gray values of the original scenes according to the epipolar geometry resulting in normalized scenes.

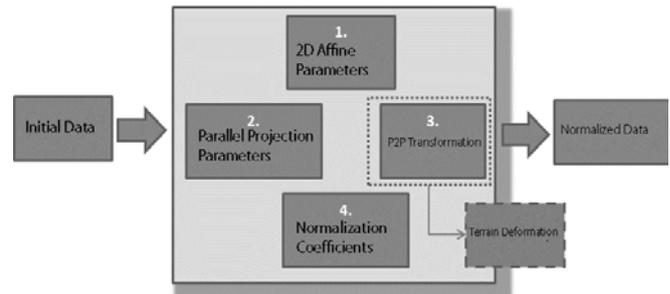


Figure 3. Resembling the satellite recordings in the geometry of the epipolar straight line

IV. DIGITAL TERRAIN MODELLING BASED ON SATELLITE IMAGERY

Digital terrain modelling is of great importance for a wide variety of applications such as the generation of orthophotographs, object recognition, urban modelling and creating perspective views. Digital satellite recordings taken from platforms with high resolution recording sensors (e.g., SPOT-5, IKONOS, QUICKBIRD, ORBVUE, and EOS-1) as well as digital photogrammetric recordings taken from aerial digital cameras (e.g., Z / I DMC, Vexcel UltraCamXp, HRSC, ADS40 and ADS80) are an excellent source for efficient, economical and accurate generation of the terrain numerical model. In general, the process of generating the numerical model can be described as follows [8]:

- the selection of details in one of the images that form a stereo: the selected details must be characteristic in the recording or in object space;
- identification of the conjugated elements in the second image: this problem is known in photogrammetry as the correlation/ correspondence establishment;
- space intersection: conjugate points are used in a space intersection process to determine the corresponding points in the object space. This process is based on the mathematical model between the registration coordinates and the field coordinates;
- thickening of the points: is made by interpolation.

The most difficult problems associated with generating the numerical model of the terrain are the correlation and the establishment of the mathematical model that describes the relationship between the image coordinates and the object

coordinates. In order to obtain a correct solution in the case of the correlation problem, prominent details represented by a characteristic signal are usually used [8].

Limiting the search space, the region where the conjugate points may exist, is another aspect that can be used to reduce ambiguity. Generation of standardized records (i.e., reassembly of images in epipolar geometry) is the common approach for limiting the search space for identifying conjugated points [7]. In the normalized images the conjugate points are along the same epipolar lines for the common coverage areas.

The mathematical model that describes the relationship between image coordinates and object coordinates corresponding to the conjugate points can be expressed using rigorous or approximate modeling of perspective geometry associated with the recording system.

Rigorous modeling requires accurate knowledge of image geometry; inner and outer orientation parameters are explicitly implicated [2]. Determining these features requires control information that can be in the form of support points, and/or navigation units (GPS/INS). However, determining these parameters can be hindered by the lack of sufficient control information, poor image geometry (e.g., narrow-angle sensors) and/or by the provider's secrecy of these parameters (e.g., Space Imaging does not provide parameters for interior and exterior orientation for IKONOS recordings).

That is why there has been and there is an increased interest within the photogrammetry community for the adoption of approximate models, their feature being that such models do not imply any exact knowledge of the image geometry, or of the internal and external characteristics of the sensor.

Of all these models, the parallel projection has gained popularity due to the simplicity and precision of the representation of the perspective projection characteristic of the recordings taken with linear sensors with a narrow angle of view that move with constant speed and orientation.

Another reason for using the parallel projection is the direct way of generating the normalized records, a necessary step for increasing the reliability and reducing the search space specific to the image correlation process.

For systems moving at constant speed and orientation, the image geometry can be described by a perspective projection along the scan direction and a parallel projection along the path of travel along the trajectory.

Furthermore, for narrow-angle systems the image geometry along the scan direction may be approximated by a parallel projection.

The constant trajectory and narrow viewing angle associated with the high-resolution recording platforms provided with linear sensors (e.g., the IKONOS viewing angle is less than one degree) directly contribute to the validation of the parallel projection as the best model to describe the mathematical relationship between image and object coordinates [1, 2, and 4].

## V. EXPERIMENTS

For evaluating the performance of the parallel projection model, two SPOT level 1A panchromatic recordings were used (as seen in Figure 4 and Figure 5).

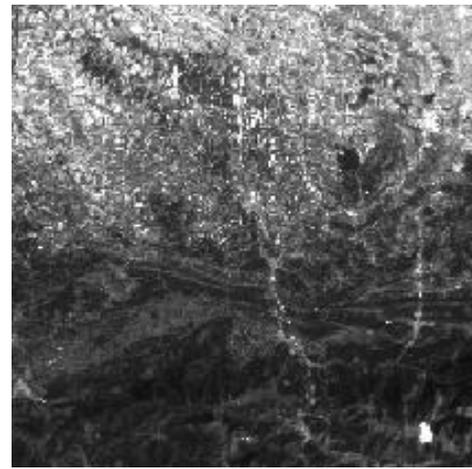


Figure 4. Spot Level 1a Panchromatic Left Image

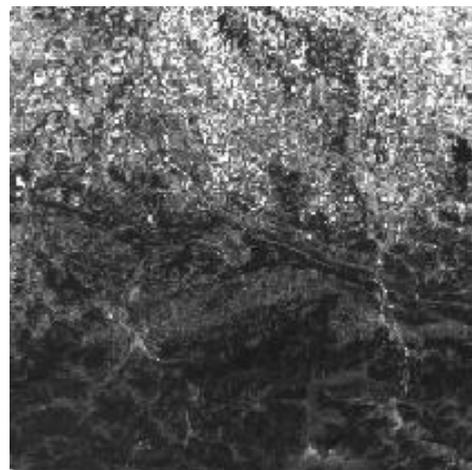


Figure 5. Spot Level 1a Panchromatic Right Image

The main technical characteristics of the two recordings are presented in Table I.

TABLE I. TECHNICAL CHARACTERISTICS OF THE SPOT DATASET

Parameter	Image 01	Image 02	Obs.
K-J Id	043-263	042-263	Interval = 26 day B/H = 0.87 Coverage: ~75%
Record date	21.08.99	26.07.99	
Focal distance (mm)	1082	1082	
Angle of incidence	+ 20.6 grade	- 26.2 grade	
Angle of orientation	14.5 grade	8.7 grade	
Pixel (mm)			
Pixel dimension (m)			
# rows and columns			
Principal point coordinates	N043°05'14" E001°35'28"	N043°05'07" E001°22'53"	

In experiments 10 support points were used for indirect estimation of the parameters corresponding to the nonlinear shape of the projection pairs.

The values of these parameters are presented in Tables II and VII control points respectively for estimating the transformation accuracy.

TABLE II. 2D AFFINE PARAMETERS ESTIMATED INDIRECTLY

	Left image		Right image	
	Without trend	With trend	Without trend	With trend
$\sigma$ , pixels	1.14	1.14	1.02	1.02
$\psi$	-8.45E-002	-8.48E-002	-4.86E-002	-4.85E-002
$A_1$	8.527E-002	8.527E-002	8.031E-002	8.031E-002
$A_2$	-2.650E-002	-2.650E-002	-1.856E-002	-1.856E-002
$A_3$	6.240E-002	6.240E-002	-7.153E-002	-7.153E-002
$A_4$	4.372E+003	4.372E+003	-5.667E+003	-5.667E+003
$A_5$	2.593E-002	2.593E-002	1.628E-002	1.628E-002
$A_6$	9.621E-002	9.621E-002	9.889E-002	9.889E-002
$A_7$	-1.834E-004	-1.852E-004	-1.966E-004	-1.963E-004
$A_8$	-8.486E-002	-8.451E-002	-1.882E+005	-1.882E+005

These parameters were used to normalize the two SPOT records.

The result of the reassembly in the geometry of the epipolar straight line is shown in Figure 6.

The two standardized records are superimposed to generate a stereo image using the anaglyphs method which can be stereo visualized using a pair of anaglyph glasses.

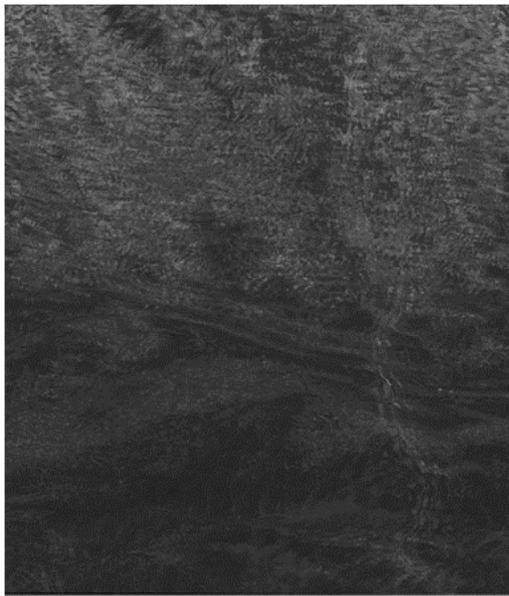


Figure 6. Stereo Image with Anaglyph of Standardized Recordings

## VI. CONCLUSION

In this paper, the methodology of reassembling the satellite recordings in the geometry of the epipolar straight line was presented using the parallel projection model. This methodology has two important characteristics.

First of all, the parallel projection is a mathematical model of establishing the correspondence between the coordinates of the recording and the coordinates of the ground for recordings taken with sensors installed on satellite platforms with a narrow angular field of view (AFOV), as is the case of high resolution satellite sensors.

Parallel projection involves very few parameters that can be determined using a limited number of ground control points (GCPs), and does not require knowledge of the interior and exterior features of the retrieval system.

Secondly, this model can be used to reassemble the satellite recordings in the epipolar geometry, obtaining normalized records.

This geometrical processing is very important for the geolocation of the digital terrain model, since it reduces the space of searching the conjugate points to a single dimension.

Experimental results using SPOT satellite recordings support the feasibility of the method.

## APPENDIX A. LINEARIZED FORM OF PERSPECTIVE TO PARALLEL PROJECTION TRANSFORMATION

The transformation from perspective projection to parallel projection along the scan line is given by (2).

In (2)  $y$  represents the  $y$  coordinate in perspective geometry, a coordinate that can be measured directly within the scene. The parameters of the blue 2D transformation along with the longitudinal tilt angle can be determined by the method of least squares if we have more than five control points. Such a method of determination involves linearizing (2) and using approximate values.

The linearization of the equation implies the Taylor series development around neighborhoods close to the true values, called in the process of compensating approximate values.  $A_5, A_6, A_7, A_8$  and  $\psi$  are unknown parameters, so to linearize the equation we have to calculate the partial derivatives with respect to these unknowns around their approximate values, as follows:

### A. The linear form without the trend information of the relief

$$\left(\frac{\partial y}{\partial A_5}\right)_0 = \frac{\frac{\partial f}{\partial A_5} g - f \frac{\partial g}{\partial A_5}}{g^2}$$

$$\left(\frac{\partial y}{\partial A_6}\right)_0 = \frac{\frac{\partial f}{\partial A_6} g - f \frac{\partial g}{\partial A_6}}{g^2}$$

$$\left(\frac{\partial y}{\partial A_7}\right)_0 = \frac{\frac{\partial f}{\partial A_7} g - f \frac{\partial g}{\partial A_7}}{g^2}$$

$$\left(\frac{\partial y}{\partial A_8}\right)_0 = \frac{\frac{\partial f}{\partial A_8} g - f \frac{\partial g}{\partial A_8}}{g^2}$$

$$\left(\frac{\partial y}{\partial \psi}\right)_0 = \frac{\frac{\partial f}{\partial \psi} g - f \frac{\partial g}{\partial \psi}}{g^2}$$
(4)

where:

$$f = A_5 X + A_6 Y + A_7 Z + A_8$$
(5)

$$g = 1 + \frac{\tan(\psi)}{c} (A_5 X + A_6 Y + A_7 Z + A_8)$$

### B. The linear form with the trend information of the relief

In order to introduce in the model the information on the relief trend, we start from (2) and we quantify the error due to the working hypothesis that the terrain is flat; this correction is given by the equation:

$$\Delta y = s \Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)}$$
(6)

From the analysis and observations made in the previous section, the following form of the equation for transforming the perspective projection into parallel projection is taken into account, also considering the trend of the relief:

$$y = \frac{A_5X + A_6Y + A_7Z + A_8 + s\Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)}}{1 + \frac{\tan(\psi)}{c} \left( A_5X + A_6Y + A_7Z + A_8 + s\Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)} \right)} \quad (7)$$

Partial derivatives of the equation in relation to the unknowns  $A_5, A_6, A_7, A_8$  and  $\psi$  around the approximate values are given by the relations:

$$\left( \frac{\partial y}{\partial A_5} \right)_0 = \frac{\frac{\partial f}{\partial A_5} g - f \frac{\partial g}{\partial A_5}}{g^2}$$

$$\left( \frac{\partial y}{\partial A_6} \right)_0 = \frac{\frac{\partial f}{\partial A_6} g - f \frac{\partial g}{\partial A_6}}{g^2}$$

$$\left( \frac{\partial y}{\partial A_7} \right)_0 = \frac{\frac{\partial f}{\partial A_7} g - f \frac{\partial g}{\partial A_7}}{g^2}$$

$$\left( \frac{\partial y}{\partial A_8} \right)_0 = \frac{\frac{\partial f}{\partial A_8} g - f \frac{\partial g}{\partial A_8}}{g^2}$$

$$\left( \frac{\partial y}{\partial \psi} \right)_0 = \frac{\frac{\partial f}{\partial \psi} g - f \frac{\partial g}{\partial \psi}}{g^2}$$

where:

$$f = A_5X + A_6Y + A_7Z + A_8 + s\Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)} \quad (9)$$

$$g = 1 + \frac{\tan(\psi)}{c} \left( A_5X + A_6Y + A_7Z + A_8 + s\Delta Z \frac{\sin(\alpha)}{\cos(\psi + \alpha)} \right)$$

APPENDIX B. THE RESULTS OF THE REASSEMBLY PROCESS IN THE GEOMETRY OF THE EPIPOLAR STRAIGHT LINE. SATELLITE RECORDS FROM THE TOULOUSE TEST AREA

C. Indirect determination of 2D affine parameters corresponding to the two records

TABLE III. LEFT SPOT LEVEL 1A PANCHROMATIC IMAGE

Values/Parameters	Approx.	Iteration 1	Iteration 2	Iteration 3
A <sub>1</sub>	0.085279	-	-	-
A <sub>2</sub>	-0.026504	-	-	-
A <sub>3</sub>	0.062403	-	-	-
A <sub>4</sub>	4372.997794	-	-	-
A <sub>5</sub>	0.025897	0.025906	0.025933	0.025933
A <sub>6</sub>	0.096146	0.096084	0.096218	0.096218
A <sub>7</sub>	0.000265	-0.000313	-0.000183	-0.000183
A <sub>8</sub>	-188774.602147	-188665.826001	-188922.589801	-188924.066004
$\psi$ (grade)	4.2358	4.2351	4.2372	4.2381
$\sigma$	1.1456			

TABLE IV. RIGHT SPOT LEVEL 1A PANCHROMATIC IMAGE

Values/Parameters	Approx.	Iteration 1	Iteration 2	Iteration 3
A1	0.080317	-	-	-
A2	-0.018568	-	-	-
A3	-0.071536	-	-	-
A4	-5667.160894	-	-	-
A5	0.016275	0.016268	0.016288	0.016288
A6	0.098861	0.098688	0.098897	0.098898
A7	0.000098	-0.001198	-0.000206	-0.000196
A8	-188164.200126	-187847.164748	-188236.323967	-188238.606906
$\psi$ (grade)	-6.4840	-6.4247	-6.4435	-6.4449
$\sigma$	1.0219			

D. Derivation of the parameters of the parallel projection

TABLE V. PARALLEL PROJECTION PARAMETERS

Parameter	Left image	Right image
L	-0.552894	0.647751
M	0.150581	-0.105180
N	0.819532	0.754556
S	0.099344	0.099569
Omega	1.493370	-4.089645
Phi	-11.550248	17.762809
Kappa	-14.497419	-7.745730
Dx	4372.997794	-5667.160894
Dy	-188924.066004	-188238.606906

E. Normalization parameters

It represents the coefficients of the corresponding transformation corresponding to each record: A-scale in the x direction, B, D-rotations around the axes Ox respectively Oy, C, F-translations in the x and y direction and E-scale in the y direction.

TABLE VI. AFFINE TRANSFORMATION COEFFICIENTS

Parameter	Normalization plane	Parameter	Normalization parameters	
			Left image	Right image
Omega	0	A	1.112787	1.207756
Phi	0	B	0.093351	0.019357
Kappa	-11.902171	C	12123.049065	9841.243456
Dx	-647.081550	D	-0.061866	0.053784
Dy	-188581.336455	E	0.9943893	0.994123
S	0.099457	F	-446.718576	-1144.085684

F. Precision of transformation

TABLE VII. MODEL WITHOUT USING INFORMATION ABOUT THE RELIEF TREND

Cross-sectional parallax	Deviations of field coordinates (X, Y, Z)				
	X	Y	Z		
-0.033836	-4.353849	2.602578	8.354270		
-0.579681	8.935576	-3.100562	-8.257761		
-0.321773	-2.982916	-4.633419	5.970660		
-1.101824	-2.484635	-5.279810	-10.950812		
-1.227278	-2.925766	2.466472	7.450697		
-1.863461	-1.051631	1.995013	9.469098		
-1.393756	2.108424	-4.050284	-2.036153		
Media	Std. dev.	Media XY	Std. dev. XY	Media	Std. dev.
-0.931659	0.645506	-0.981139	4.221205	1.428571	8.451730

TABLE VIII. MODEL USING THE INFORMATION ON THE RELIEF TREND

Cross-sectional parallax		Deviations of field coordinates (X, Y, Z)			
		X	Y	Z	
-0.033784		-4.354553	2.600784	8.354180	
-0.581596		8.935246	-3.101524	-8.257844	
-0.327946		-2.981167	-4.628815	5.970940	
-1.111615		-2.485855	-5.282984	-10.950978	
-1.237953		-1.050677	2.469012	7.450829	
-1.879012		-0.1710618	1.994762	9.469082	
-1.411419		2.108016	-4.051402	-2.036219	
<b>Media</b>	<b>Std. dev.</b>	<b>Media XY</b>	<b>Std. dev. XY</b>	<b>Media</b>	<b>Std. dev.</b>
<b>-0.940475</b>	<b>0.651472</b>	<b>-0.714301</b>	<b>3.966297</b>	<b>1.428570</b>	<b>8.451817</b>

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