An Overview of Signal Processing Methods for Signal Source Localization

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***Abstract*—The majority of modern applications require signal processing methods that can provide the expected results, such as signal source's position in space, based on the time delay between the received signals. The performance of each method varies according to the signal to noise ratio, the sampling frequency, or the type of signal, etc. This paper aims to present and compare several signal processing methods in an effort to determine the optimum solution for the localization of signal sources. Methods such as the Spectrogram, Wavelet Transform, Cross-Correlation or Recurrence Plot Analysis were simulated using Matlab software. The conclusion is that Cross-Correlation is the optimum method in real-time applications, in the case of a high sampling frequency, followed by Recurrence Plot Analysis.**

***Index Terms*—signal, sampling frequency, time domain, time delay, recurrence, detection curves, time of arrival**

I. Introduction

A signal can be a series of values observed over time, an electrical signal, such as voltage or current, an electromagnetic wave carrying useful information from the source to destination, etc. A signal can be either analog or digital, either having a mathematical expression or being non-deterministic.

A digital signal is obtained after an analog to the numeric conversion of a continuous-time signal, which requires sampling and quantization of the analog signal and its transformation into a digital.

Sampling a signal means recording signal values continuously, at discrete points over time, at exact intervals, equal to the inverse sampling frequency. The sampling frequency is the number of samples in a period, it complies with the Nyquist principle which states that the sampling frequency must be greater than, or equal, to twice the frequency of the signal [1].

Signal processing methods can be used for signal characterization, signal classification, signal source localization, and many more.

To localize a signal source, the delays between the arriving signals need to be known, so that Time Difference of Arrival (TDOA) or Time of Arrival (TOA) can be applied. A source localization system, capable of providing the coordinates of a signal source, usually consists of a network of microphones or receivers and acquisition software.

In this paper, TDOA method is used for signal source`s position estimation. To estimate the position of a signal source in space, first the signal needs to be detected, the time of arrival (TOA) to be computed, or the moments each microphone receives the signal. To find the best solution for signal detection, a comparison between signal processing methods is done.

In order to simulate the system's behavior, four ideal signals are generated, and then is applied a signal to noise ratio of 5 dB.

The signal has a frequency of 4 kHz, time duration of 55 ms, being sampled at a 44100 Hz sampling frequency.

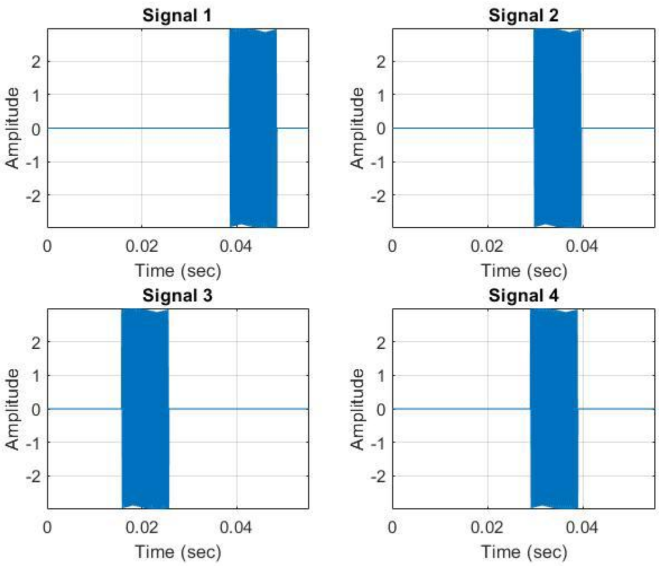

Figure 1. Ideal signals in the time domain

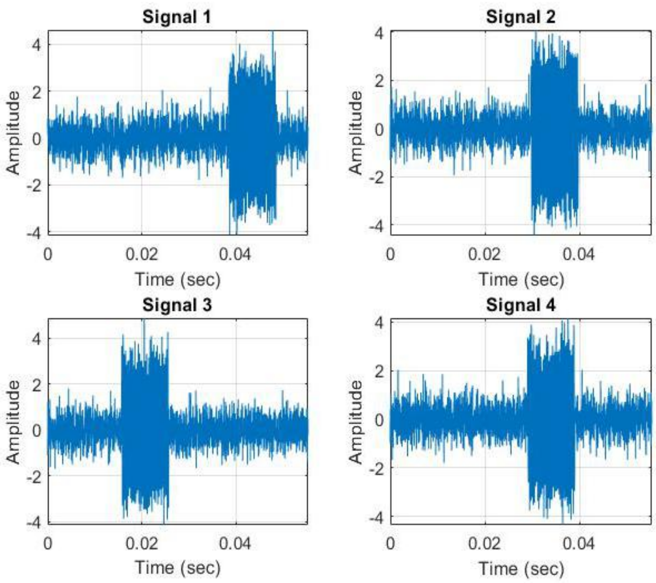

Figure 2. Signals with a SNR =5 dB in the time domain

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II. Signal Processing Methods

A. The Cross-Correlation

The Cross-Correlation function provides information on the similarity of two signals and is dependent on the relative delay of one signal to the other.

The cross-correlation function is mathematically expressed as follows:

$$R_{s1,s2}(k) = \sum_{N=-\infty}^{\infty} s1(n)s2(n-k), k = 0, \pm 1, \pm 2, ... \quad (1)$$

Cross-correlation can be used to determine the degree of similarity of the signals and the delays in samples, subsequently expressed in seconds. The greatest resemblance will be given by the point at which the correlation function will be maximum.

The delay can be calculated by the difference between the sample index for the first moment of time at which the likeness between the signals is maximum.

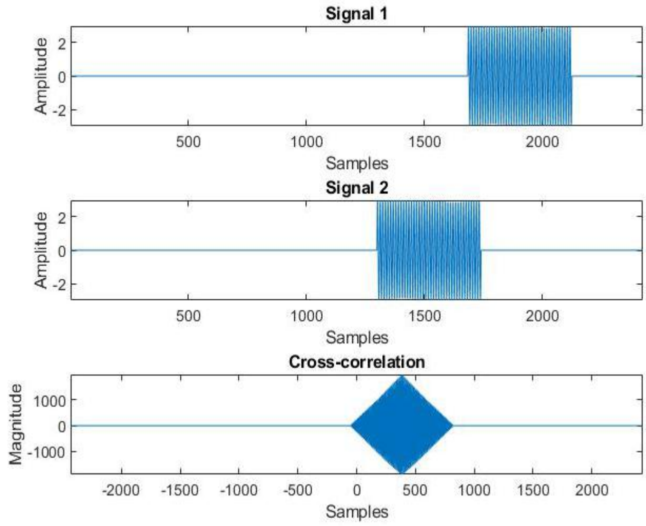

Figure 3. Graphical representation of the signals and Cross-correlation

Time moments can be calculated by dividing the sample index by the sampling frequency.

The difference of time arrival estimation is done by subtracting the arrival times for the 4 signals from the arrival time chosen as the t_1 reference:

TABLE I. The Time Delays between Signals

The time delays between the ideal signals (seconds)	The time delays between the noisy signals(seconds)
$\Delta_{12} = t_1 - t_2 =$ 0.0087	$\Delta_{12} = t_1 - t_2 =$ 0.0087
$\Delta_{13} = t_1 - t_3 =$ 0.0228	$\Delta_{13} = t_1 - t_3 =$ 0.0228
$\Delta_{14} = t_1 - t_4 =$ 0.0095	$\Delta_{14} = t_1 - t_4 =$ 0.0097

Cross-correlation is an easy-to-implement, fast and efficient method, for determining the similarity between two signals, thus the time delay. It does not involve large computing resources and can work with large signals. The cross-correlation depends on the sampling frequency and is noise robust.

The disadvantage would be that in case of a small sampling frequency, the resolution in time would degrade, small time delays (ns, ms) would not be captured. This method is recommended for real-time applications.

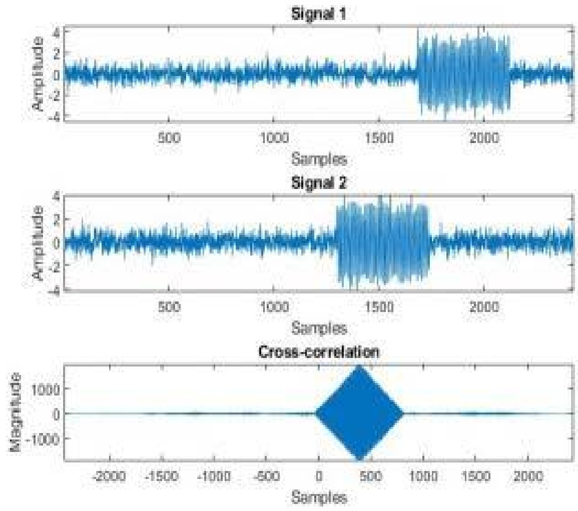

Figure 4. Graphical representation of the signals and the Cross-correlation

B. The Spectrogram

Long-term digital signals (recorded songs, for example) are processed on sample sections, both to play better analysis over time and frequency, and for the speed of signal processing.

The spectrum of a signal is a representation of the energy distributed in a frequency range.

The spectrogram is based on the Short-Time Fourier Transform (*STFT*), which can extract both information on the frequency components and time information. *STFT* consists in multiplying the analyzed signal with a window signal, then applying the Fourier Transform to successive portions of it [3].

To calculate the spectrogram of a signal, it is divided into blocks of equal lengths, superimposed on a specified number of samples, on which the Fourier Transform is run. *STFT* can be mathematically rendered as [2]:

$$STFT(\tau, f) = \int_{-\infty}^{\infty} x(t) g^*(t-\tau) e^{-j2\pi ft} dt \quad (2)$$

The performance of the *STFT* analysis depends on the chosen window function. It should be noted that the resolution in frequency is better the larger the window size, but in this case, the resolution over time will decrease. Heisenberg's principle is respected, which states that: "the product between the time and frequency durations of a signal is limited by a non-null value".

When the absolute value of the *SFTF* is represented, we get the spectrogram [2]. A spectrogram is a graphical view of the time-frequency range of a signal, where you can track the variation of the energy of a signal over time.

The spectrogram is a three-dimensional graph, its axes being: the axis of time, the axis of frequencies, and the axis of amplitude or spectral power. Most of the time, to simplify the interpretation, it is represented as two-dimensional (time-frequency), the components of amplitude or spectral power being rendered by the intensity of the colors.

As practical applications, it can be used in the field of acoustics, seismology, etc.

For the spectrogram a 64 size window was used, the number of overlapping points was 50, and the number of points at which the *STFT* is performed was 128.

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To calculate the difference time of arrival, a column summary of the values corresponding to the frequencies in the signal is computed. The time of arrival corresponds to the index of the first peak of a 40% from the maximum amplitude in the detection curve. This is necessary for the detection of the signal status transition. The chosen threshold influences the first peak detection (the first transition in the signal). If this threshold is not well chosen, the algorithm would not detect the first transition in the signal, so the result would not correspond to the arrival time.

Following the visualization of the spectrograms of non-ideal signals, small energy accumulations are shown at different times, i.e. new frequency components, due to SNR of 5 dB.

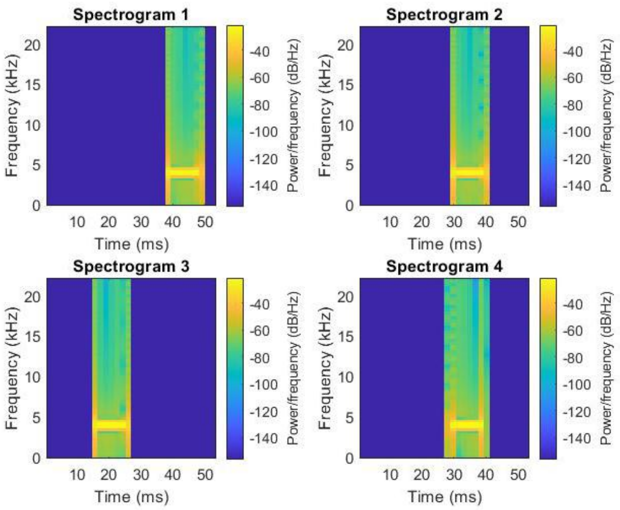

Figure 5. The spectrograms of the ideal signals

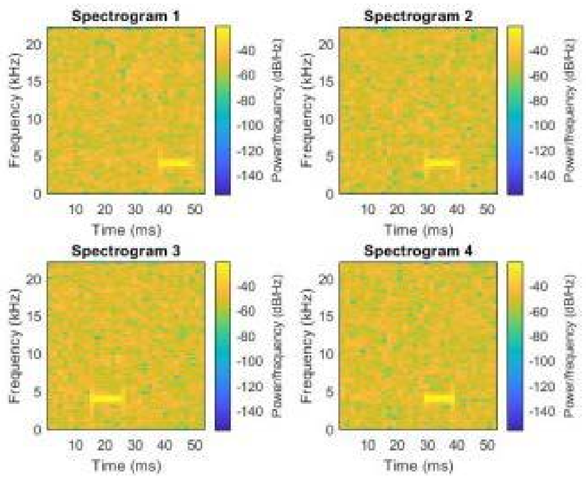

Figure 6. The spectrogram of the noisy signals

TABLE II. THE TIME DELAYS IN CASE OF THE SPECTROGRAM

The time delays between the ideal signals (seconds)	The time delays between the noisy signals (seconds)
$\Delta_{12} = t_1 - t_2 =$ 0.0091	$\Delta_{12} = t_1 - t_2 =$ 0.0091
$\Delta_{13} = t_1 - t_3 =$ 0.0234	$\Delta_{13} = t_1 - t_3 =$ 0.0234
$\Delta_{14} = t_1 - t_4 =$ 0.0098	$\Delta_{14} = t_1 - t_4 =$ 0.0098

Less accuracy than in the case of Cross-correlation is observed. Small numerical errors might be caused by mathematical calculations when performing *STFT*, window choice, number of overlap points, i.e. actual calculation of the spectrogram. Also, when using the Spectrogram, the results will be affected due to the energy of the noise present throughout the signal duration.

The advantages of the Spectrogram would be that it is a noise-robust method and is better used in case of short impulses, or short-time signals, which have a large spectrum, otherwise, the frequency resolution would be below expectations. It also depends on the sampling frequency, the parameters used for spectrogram generation.

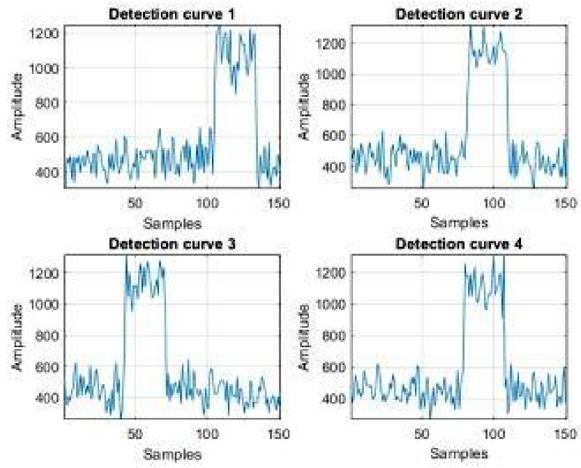

Figure 7. Detection curves for ideal signals in the case of the Spectrogram

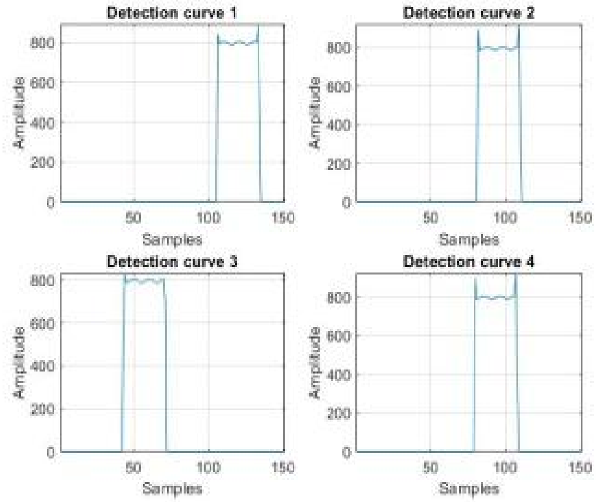

Figure 8. Detection curves for noisy signals in the case of the Spectrogram

The disadvantage of using this method would be the performing of *STFT*, because it could provide numerical errors. More than that, the Heisenberg principle should be respected so that an equilibrium between time resolution and frequency resolution is kept.

C. *The Wavelet Transform*

Wavelet analysis is a relatively new method of signal analysis used in various fields such as engineering, science, medicine, and finance. Unlike the Fourier Transform that decomposes the signal into amounts of sines and cosines, the Wavelet Transform breaks down the signal in scale and time, given by the wavelet function.

A wavelet function can be considered the shortest "oscillation waveform" [4]. Wavelet analysis is calculated on different time intervals, resulting in a two-dimensional representation called a scalogram.

A wavelet function $\psi(t)$ satisfies the following conditions [5]:

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i) The signal's energy is concentrated, for the most part, in a finite time frame:

$$E = \int_{-\infty}^{\infty} \left|\psi(t)\right|^2 dt < \infty \tag{3}$$

ii) The wavelet function has no continuous component, it has no zero-frequency component. If the Fourier Transform of the wavelet function $\psi(t)$, the following condition, called a condition of admissibility, must be fulfilled:

$$C_{\psi} = \int_{0}^{\infty} \frac{\left|\psi(f)\right|^2}{f} df < \infty \tag{4}$$

iii) For complex wavelet functions the Fourier transform $\psi(t)$ must be real and positive.

The Wavelet Transform can be mathematically defined as a signal resulted from the inner product of the analyzed signal and the wavelet signals [3]:

$$WT_X^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} x(t) \psi^* \left(\frac{t-\tau}{s}\right) dt \tag{5}$$

The "mother wavelet" is defined as:

$$\psi_{(\tau,s)}(t) = \frac{1}{\sqrt{|s|}} \psi\left(\frac{t-\tau}{s}\right) \tag{6}$$

where $\psi(t)$ represents the transformation function, τ is a real number, and $s > 0$, representing the scale parameter of the mother wavelet function. The term $\frac{1}{\sqrt{|s|}}$ is a normalization factor that assures that the energy of the wavelet function $\psi_{(\tau,s)}(t)$ remains constant. "*" representing the complex conjugation operation.

By changing the scale parameter *s*, the parent wavelet function contracts $(0 < s < 1)$ and expands $(s > 1)$ causing both changes of the central frequency and the window size [3].

A wavelet function has its central frequency f_c for each value of the parameter *s*, *s* being inversely proportional to the f_c.

The product $\Delta t \cdot \Delta f$ being constant, by decreasing its scale parameter, the resolution over time will increase, which will lead to a lower resolution in frequency. The inverse is valid [3].

By changing the τ parameter, the wavelet function can be moved along the signal, providing information on the time moments when the frequency components appear.

The elements of the function $WT_X^{\psi}(\tau, s)$ are called wavelet coefficients, where each coefficient is assigned a frequency and a time moment [5]. The resulting wavelet coefficients will form the wavelet series. Continuous Wavelet Transform involves intercropping the analyzed signal with a set of translated and scaled versions of the same wavelet function [5].

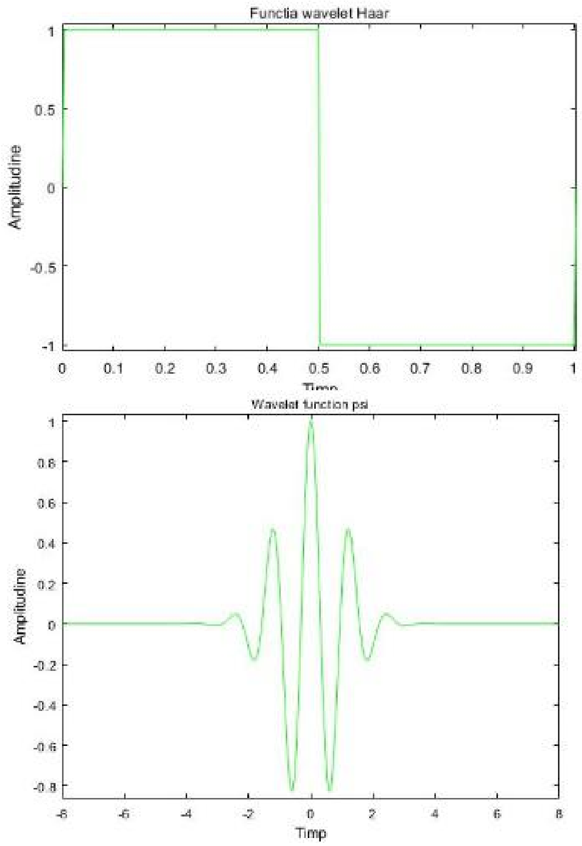

Figure 9. Graphical representation of Haar Wavelet and Morlet Wavelet in the time domain

Wavelet signals are formed by changing the parameters of scale *s* and τ, starting from the main function "mother wavelet" – the parent function [5]. The wavelet function is chosen according to the application, depending on the results to be achieved. A set of functions used in the calculation of the wavelet transform:

1. The Haar Wavelet Function:

$$\psi_H(t) = \begin{cases} 1, t = \left[-\frac{1}{2}, 0\right] \\ -1, t = (0.1, 2] \\ 0, \ \text{otherwise} \end{cases} \tag{7}$$

2. Morlet Wavelet Function:

$$\psi(t) = \frac{1}{\sqrt[4]{\pi}} e^{\frac{-t^2}{2}} \cos(\pi t) \tag{8}$$

The scalogram is a graphical representation of the square of the Continuous Wavelet Transform module. It respects, as in the case of a spectrogram, the Heisenberg principle concerning the time and frequency resolution [5].

The time delays estimation using the Wavelet Transform is almost identical as in the case of the Spectrogram. The difference is that the Wavelet Transform provides more information about the time when the frequency components appear. By the variation of the s scale, better temporal resolution can be achieved.

Some strips appear at different times within the computed scalogram for the noisy signals, corresponding to lower energy values, which denote the presence of noise in the signal and its very low energy. The signal is "weakened" by the noise.

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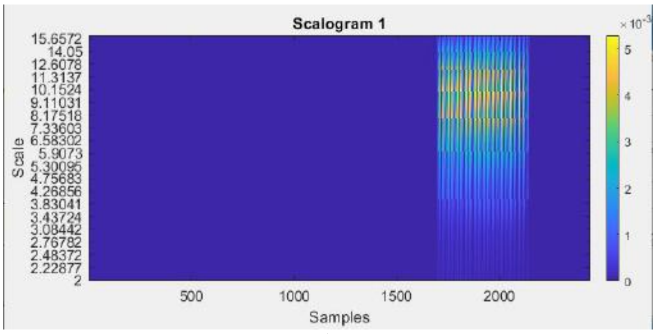
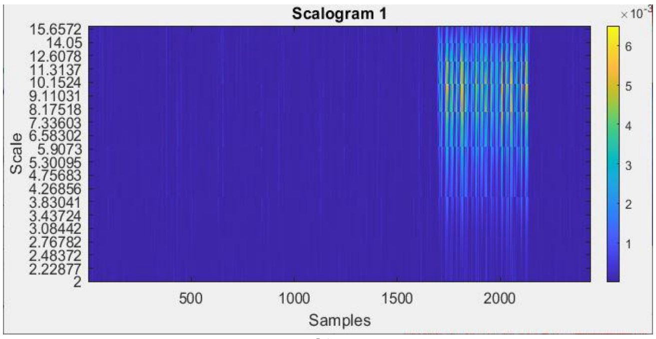


a)



b)

Figure 10. a) First ideal signal in the time domainand its scalogram using Haar Wavelet b) Noisy signal in the time domain and its scalogram using Haar Wavelet

The graphical representation of the detection curves for the scalograms shows the impact of noise on the signals, which is why threshold of 40% from the maximum amplitude peak is considered to detect the transition, i.e. finding the sample corresponding to the time of arrival.

The deterioration of the detection curves in the case of noise-affected signals by the appearance of additional frequency components and the modification of wavelet coefficients is observed. Much smaller variations in the time delays than in the case of Spectrogram are observed due to the better temporal resolution of the Wavelet Transform.

TABLE III. THE TIME DELAYS IN CASE OF THE WAVELET TRANSFORM

The time delays between the ideal signals (seconds)	The time delays between the noisy signals (seconds)
$\Delta_{12} = t_1 - t_2 = 0.0087$	$\Delta_{12} = t_1 - t_2 = 0.0088$
$\Delta_{13} = t_1 - t_3 = 0.0228$	$\Delta_{13} = t_1 - t_3 = 0.0228$
$\Delta_{14} = t_1 - t_4 = 0.0095$	$\Delta_{14} = t_1 - t_4 = 0.0095$

Similar to the Spectrogram, the same characteristics are valid. The advantage in this case would be a better resolution in time and frequency, due to the wavelet function that is used.

It is important both to choose the right wavelet function and to set the right scale and time parameters, because it influences both the central frequency of the function and the wavelet coefficients, thus affecting the frequency resolution of the scalogram.

D. Recurrence Plot Analysis

Recurrence is a fundamental property of dynamic systems that can be exploited to characterize the behavior of the system in the State Space [6]. The recurrence chart, introduced in 1987, is a powerful tool for viewing and analyzing recurrences [15].

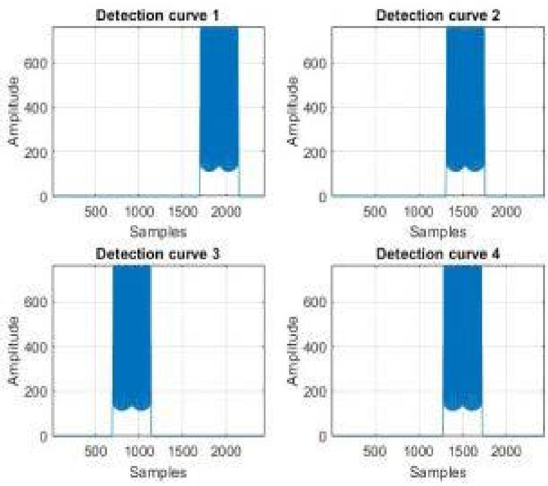


Figure 11. Detection curves of ideal signals in the case of Wavelet Transform

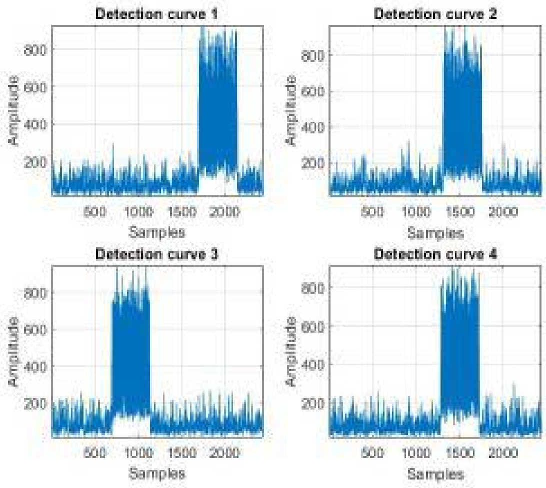


Figure 12. Detection curves of ideal signals in the case of Wavelet Transform

E. Recurrence Plot Analysis

Recurrence is a fundamental property of dynamic systems that can be exploited to characterize the behavior of the system in the State Space [6]. The recurrence chart, introduced in 1987, is a powerful tool for viewing and analyzing recurrences [15].

The first step towards recurrence analysis is the application of a technique known as State-Space Embedding applied to each sample [6]. The process consists of converting each sample of the signal into a vector shape, the dimensions of which are given by a specific parameter. Each vector is considered a state [6], representing a point in a multidimensional space. If the actual phase space is m-dimensional, then the size of the encapsulation space must have no more than $2m+1$ dimensions for the system dynamics to be fully captured, in other words, for the details of the attractor to be captured [7].

1) The State-Space

The states of a natural or technical system change over time. The study of such a complex dynamic system is an important task in various scientific fields. Understanding, describing, and predicting changes is of great importance in everyday life, for example in predicting weather, earthquake occurrence, etc.

A time series represents the amount of information had at some chronological, equidistant time about an event [8]:

$$x[n] = \{x[1], x[2], \ldots, x[N]\} \quad (9)$$

Based on time series, regressive analyses are carried out to observe the evolution of the processes and phenomena of a system. Status space elements represent possible system states to which a specific *t*-moment is assigned and are represented by *d* components. These parameters form a vector in the space of *N*-dimensional states [8]:

$$\overline{x_t} = \left(x_1(t), x_2(t), \ldots x_N(t)\right)^T \tag{10}$$

Vector $\vec{x}(t)$ defines a trajectory in the space of states. The law of evolution over time is a rule that allows the calculation of the state of the system at each time *t*, relative to the previous states. It is dependent on time and has infinite memory.

To generate the space of the states, the Time Delay Method can be used, by which the attractor or order of the space states is obtained:

$$\overline{x_i} = \sum_{j=1}^{N} x_{i+(j-1)\tau} \overline{e_j} \tag{11}$$

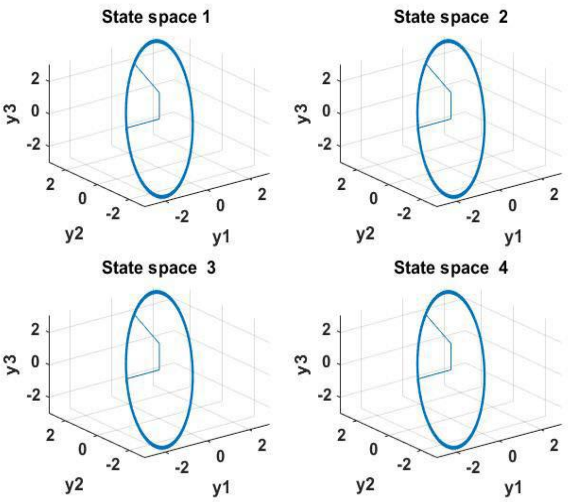


Figure 13. The State Space of ideal signals

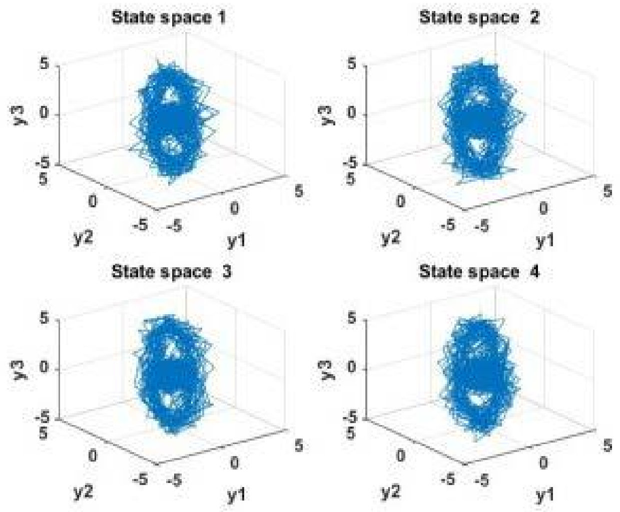


Figure 14. The State-Space of the noisy signals

The trajectory's uniformity, specific to the ideal signal (in our case a sinusoidal signal), is observed. In the representation of the reconstruction of the state space for the noisy signals, a "hedgehog" representation is captured, typical of noise-affected signals.

2) The Distance Matrix

The evolution of the system is described by a series of vectors representing the trajectory in an abstract mathematical space. Suppose $\{\vec{x}_i\}_{i=1}^{N}$ states of a system in the state space [8]. If two points *i* and *j* in the state space have a short distance between them, they can be considered the same state. Therefore, the similarity of the states between two points can be defined as a recurrence in the signal [6].

The distance matrix, constructed based on the distances between all pairs of points on the trajectory, shall be used for the trajectory representation. The representation of the matrix is independent of the position and rotation of the trajectory in its space.

The main diagonal elements will always be null, regardless of the chosen norm, the distance between a point and itself is zero [7]

$$D_{i,j} = \overline{x_i} - \overline{x_j} \tag{12}$$

where $\|\cdot\|$ represents the Euclidean distance.

The matrix of recurrences will be calculated based on the matrix of distances, as it is only an intermediate stage.

The distance matrix is the graphical representation of a square matrix with the distances between the points on the trajectory in the space of the states as elements. Each element corresponds to the distance between two points on the trajectory.

3) The recurrence matrix

The recurrence matrix is constructed based on the distance matrix, by applying a threshold. A recurrence is defined when the distance between two states *i* and *j* is less than the threshold ε, which has a very small value, i.e. the return of the trajectory in the vicinity of the points visited above [7]. The recurrence matrix is a square matrix that is based on the distance matrix, having the same dimensions:

$$R_{i,j} = \Theta\left(\varepsilon - \overline{x_i} - \overline{x_j}\right),\ i, j = \{1, 2 \ldots N\} \tag{13}$$

$R_{i,j}$ has a value of 1 when the two vectors are very close and 0 otherwise [7], where $\|\cdot\|$ is a chosen distance (Euclidean or angular distance) [9], ε represents the radius of the recurrence ball [7], and $\Theta(\cdot)$ is the Heaviside unit step function:

$$\Theta(x) = \begin{cases} 1, x \geq 0 \\ 0, x < 0 \end{cases} \tag{14}$$

The choice of threshold influences the representation and calculation of the recurrence matrix. The role of the recurrence chart is to observe the similarities between the values of the time series. Its elements are values of 0 and 1, so values lower than the chosen threshold are considered 0 and the highest is considered 1. Using the threshold ε, it can be said that the recurrence matrix is a binarization of the distance matrix, so the value 1 will be assigned to all elements below ε and 0 when they are located above this value. Binarization is the segmentation of an image into regions of interest and the removal of regions that are not important.

$$R_{i,j} = \begin{cases} 1, \overline{x_i} \approx \overline{x_j} \\ 0, \overline{x_i} \not\approx \overline{x_j} \end{cases} \tag{15}$$

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No matter the ε threshold, the recurrence matrix will have the main diagonal composed of elements with a value of 1, any point on the trajectory being neighbor to itself. The main diagonal is called identity line [7].

Hidden correlations between the elements of the analyzed time series can be captured using the recurrence diagram. The bands in the recurrence diagram represent abrupt changes in trajectory dynamics [7].

The time moments when similarities occur within the time series are observed in both the distance matrix and the recurrence matrix. In the case of the recurrence matrix, the darker points represent values of 1, i.e. a very small distance between the points, the belonging of the points of the same state is considered.

Diagonal lines appear when the trajectory returns to the same area in the state space at different times. Vertical and horizontal lines describe a period of time during which the state remains unchanged, or changes very slowly.

In the case of the analyzed signals, it is observed that the specific blue portion of a period of time during which the signal state remains the same.

In the case of the noise-affected signal, both horizontal and vertical points and lines are observed in the representation of the matrix of distances and recurrences.

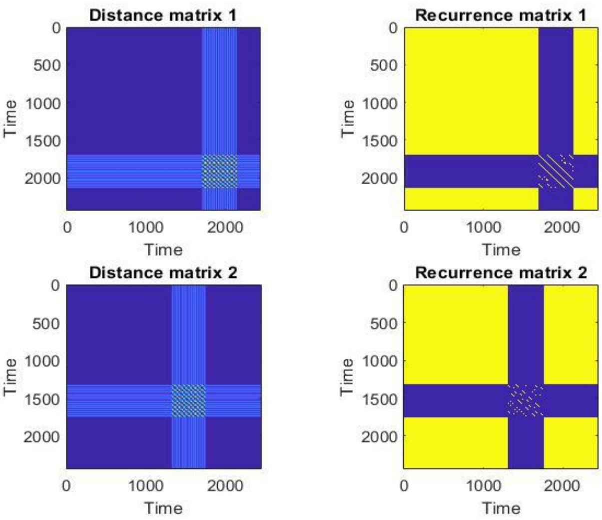

Figure 15. The distance matrix and the recurrence matrix of the ideal signals

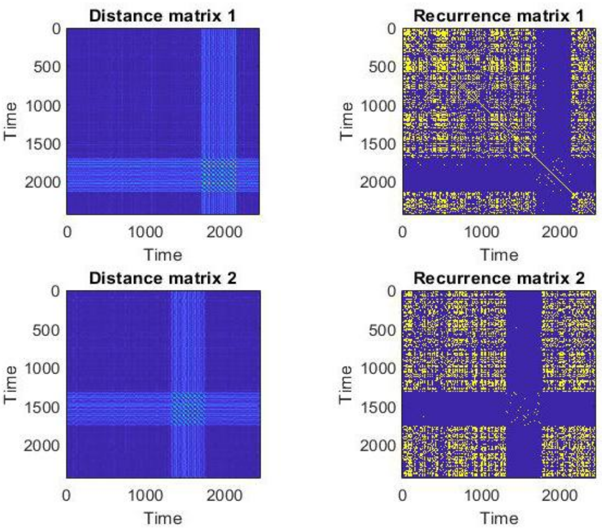

Figure 16. The distance matrix and the recurrence matrix of two of the noisy signals

TABLE IV. THE TIME DELAYS IN CASE OF RPA

The time delays between the ideal signals (seconds)	The time delays between the noisy signals (seconds)
$\Delta_{12} = t_1 - t_2 = 0.0087$	$\Delta_{12} = t_1 - t_2 = 0.0089$
$\Delta_{13} = t_1 - t_3 = 0.0228$	$\Delta_{13} = t_1 - t_3 = 0.0229$
$\Delta_{14} = t_1 - t_4 = 0.0098$	$\Delta_{14} = t_1 - t_4 = 0.0096$

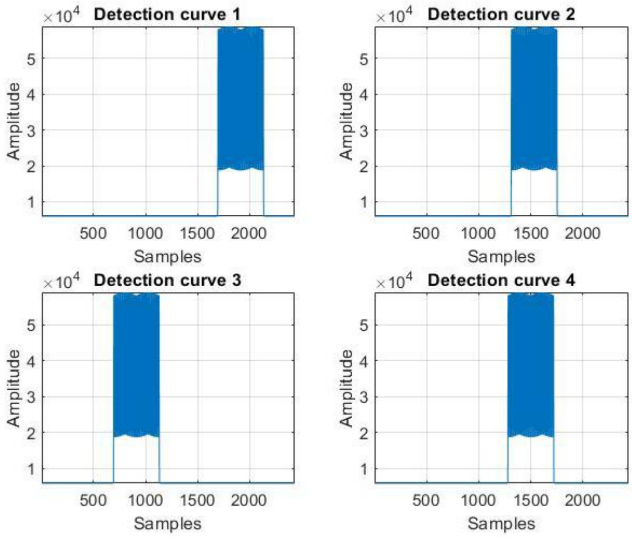

Figure 17. The detection curves of ideal signals in the case of RPA

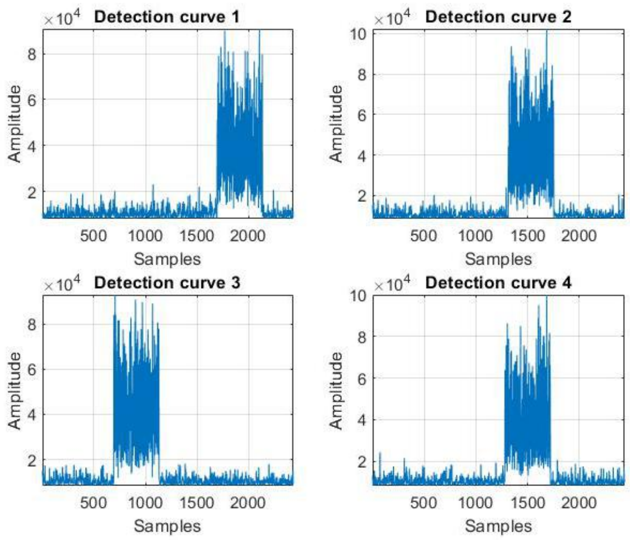

Figure 18. The detection curves of noisy signals in the case of RPA

F. *Time Difference of Arrival (TDOA)*

Time Difference of Arrival is a technique used to locate an emission source located near a network of receivers [12].

TDOA locates an acoustic source using differences of the arrival times of the signals received by a network of microphones, in other words, it represents the difference between the times that an emitted signal travels until it is received by the microphones.

An acoustic source emits a signal $s(t)$ is assumed. Signals received by 2 of the microphones are in the form of:

$$r_1(t) = \alpha s(t) + n_1(t) \quad (16)$$

$$r_2(t) = \alpha s(t-\tau) + n_2(t) \quad (17)$$

where α is the attenuation factor, $0 < t < T$, T is the observation interval, $v\tau$ is the extra distance (road difference) that the wave must cross to be received by the second microphone, relative to first microphone, where

v is the speed of the sound and the relative delay; $n_1(t)$ and $n_2(t)$ represent the added noise.

The advantage of this method is that there is no need to know the emission time. TDOA is considered as the difference between the reception times of the signal emitted by two microphones.

TDOA consists of solving a system of equations consisting of the following elements: $\{x_m, y_m, z_m\}, m = (1.M)$ the coordinates of the M microphones, and (x, y, z) the unknown coordinates of the acoustic source. $v = 343\frac{m}{s}$ is considered the speed of sound in the air at normal pressure and temperature conditions.

For a solution in two-dimensional and three-dimensional space, three microphones are required, respectively four, so a microphone is considered as a reference.

Let the $r_m = vt_m$ be the distance between the source and the microphone m. M is the total number of microphones, and $\tau_m = t_m - t_1$ the time difference for a wave to travel from the m-th microphone to the reference microphone.

It is deducted that the distances are [13]:

$$v\tau_m = vt_m - vt_1 = r_m - r_1 \tag{18}$$

resulting:

$$r_m^2 = (v\tau_m + r_1)^2 = (v\tau_m)^2 + 2v\tau_m r_1 + r_1^2 \tag{19}$$

by division with the term $v\tau_m$ and the moving to the left of the other terms:

$$0 = v\tau_m + 2r_1 + \frac{r_1^2 + r_m^2}{v\tau_m}, m = \overline{2, M} \tag{20}$$

by subtracting $v\tau_2 + 2r_1 + \frac{r_1^2 + r_2^2}{v\tau_2}$ from (20), a set of equations is obtained:

$$0 = v\tau_m - v\tau_2 + \frac{r_1^2 + r_m^2}{v\tau_m} - \frac{r_1^2 + r_2^2}{v\tau_2}, m = \overline{3, M} \tag{21}$$

r_m can be written as:

$$r_m = \sqrt{(x_m - x)^2 + (y_m - y)^2 + (z_m - z)^2} \tag{22}$$

resulting:

$$r_m^2 = x_m^2 + y_m^2 + z_m^2 + x^2 + y^2 + z^2 - 2x_m x - 2y_m y - 2z_m z \tag{23}$$

following:

$$\begin{aligned} r_1^2 - r_m^2 = x_1^2 + y_1^2 + z_1^2 - x_m^2 - y_m^2 - z_m^2 - 2x_1 x - 2y_1 y - \\ - 2z_1 z + 2x_m x + 2y_m y + 2z_m z \end{aligned} \tag{24}$$

By replacing this difference in (21):

$$\begin{aligned} 0 = v\tau_m - v\tau_2 + \frac{1}{v\tau_m}(x_1^2 + y_1^2 + z_1^2 - x_m^2 - y_m^2 - z_m^2 - \\ - 2x_1 x - 2y_1 y - 2z_1 z + 2x_m x + 2y_m y + 2z_m z) - \\ - \frac{1}{v\tau_2}(x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2 - \\ - 2x_1 x - 2y_1 y - 2z_1 z + 2x_2 x + 2y_2 y + 2z_2 z) \end{aligned} \tag{25}$$

(25) is rewritten in a lighter form:

$$0 = D_m + A_m x + B_m y + C_m z, \tag{26}$$

where:

$$A_m = \frac{1}{v\tau_m}(-2x_1 + 2x_m) - \frac{1}{v\tau_2}(-2x_1 + 2x_2) \tag{27}$$

$$B_m = \frac{1}{v\tau_m}(-2y_1 + 2y_m) - \frac{1}{v\tau_2}(-2y_1 + 2y_2) \tag{28}$$

$$C_m = \frac{1}{v\tau_m}(-2z_1 + 2z_m) - \frac{1}{v\tau_2}(-2z_1 + 2z_2) \tag{29}$$

$$\begin{aligned} D_m = v\tau_m - v\tau_2 + \frac{1}{v\tau_m}\left(x_1^2 + y_1^2 + z_1^2 - x_m^2 - y_m^2 - z_m^2\right) - \\ - \frac{1}{v\tau_2}\left(x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2\right) \end{aligned} \tag{30}$$

A set of M–2 equations in matrix form as follows:

$$\begin{pmatrix} A_3 & B_3 & C_3 \\ \vdots & \ddots & \vdots \\ A_M & B_M & C_M \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = - \begin{pmatrix} D_3 \\ D_4 \\ \cdots \\ D_M \end{pmatrix} \tag{31}$$

Solving the system by applying the Moore-Penrose pseudoreverse [14], results are obtained for (x, y, z) coordinates:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = - \begin{pmatrix} D_3 \\ D_4 \\ \cdots \\ D_M \end{pmatrix} \begin{pmatrix} A_3 & B_3 & C_3 \\ \vdots & \ddots & \vdots \\ A_M & B_M & C_M \end{pmatrix}^{-1} \tag{32}$$

The process of determining the position of a signal source in space, relative to a chosen reference system, is called localization. To obtain the results of this process, the difference of arrival times will be used, with the indication that the speed of sound in that environment is known. Some of the main reasons for using this technique is the lack of synchronization with the signal source and its efficiency in reverberant environments.

To find out the position of the source, a system of non-linear equations based on the difference of arrival times will be calculated. Ideally, a TDOA-based system is used in an environment where the following conditions are met:

- There is only one acoustic, omnidirectional source
- Multiple reflections are neglected
- There are no sources of noise in that environment
- The acoustic source is static during the acquisition
- Microphones have no noise of their own
- Changing the speed of sound due to pressure or temperature is not taken into account. The speed of sound under normal conditions in the air at room temperature is 343 m/s.
- Microphone positions are known

Following the detection of arrival times with each of the four methods implemented in the Matlab program, the non-linear system of equations is solved using differences of arrival times.

Four microphones with the following coordinates (x, y, z) expressed in meters, are considered: first microphone $(0,0,0)$, considered as reference, second microphone $(5,0,5)$, third

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microphone $(5,5,5)$ and the fourth microphone $(0,5,0)$.

The actual coordinates of the source are approximately $(7,10,5)$. A cubic room with a side of 12 meters is considered for the simulation.

The graphical representation of both the position of the microphones and the actual source and the estimated source.

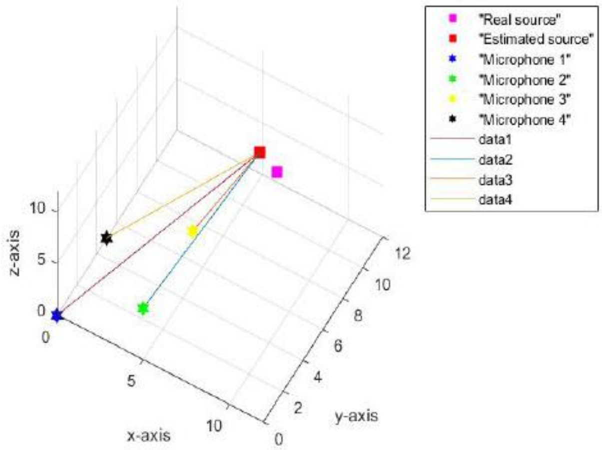

Figure 19. True position – purple color, estimated position-red color, the other colors represents the positions of the microphone

TABLE V. SOURCE ESTIMATION FOR IDEAL SIGNALS

Method	*x* axis	*y* axis	*z* axis
Real coordinates	7	10	5
Cross-correlation	6.0058	10.0137	6.0058
The Spectrogram	6.3669	10.7115	6.3669
The Wavelet Transform	6.0058	10.0137	6.0058
RPA	6.0058	10.0137	6.0058

TABLE VI. SOURCE POSITION ESTIMATIONS FOR NOISY SIGNALS

Method	*x* axis	*y* axis	*z* axis
Real coordinates	7	10	5
Cross-correlation	6.1769	10.5637	6.1769
The Spectrogram	6.3669	10.7115	6.3669
The Wavelet Transform	6.1794	10.3988	6.1794
RPA	6.2155	10.4394	6.2155

The margin of error, by subtracting the minimum and maximum values from the actual value, is:

- in the case of coordinate x : [0.6331, 0.9942] meters
- in the case of coordinate y : [–0.7115, –0.0135] meters
- in the case of coordinate z : [–1.3669, –1.0058] meters

Errors may be due to computational precision, non-linear system resolution, approximations and quantizations.

In practice, other sources of errors could be the objects in the space between the signal source and the network of microphones. Objects in the area where the purchase is made cause multipath propagation of the signal and thus errors in the location of its source. Reflections and refractions directly affect signal delays at reception; therefore time does not necessarily depend on the distance between the source and the receivers. Also, because of this, the location of the source can be “confused” with the object from which the signal was reflected. Also, the limitations of the equipment lead to errors in the detection and location of the signal source.

In a real signal source localization system, Cross-Correlation would be the best method to be used for signal processing, in the case of large signals and high sampling frequency.

For better signal source localization, Generalized Cross-Correlation, Steered-Response Power Phase Transform (SRP-PHAT), signal filtration for removing the noise, the use of more omnidirectional microphones, the use of optimizing methods such as Maximum Likelihood Estimation (MLE), and others, is recommended.

III. CONCLUSIONS

In conclusion, RPA and Cross-correlation provide the best results in detection of times of arrival for noise-affected signals, thus determining the position of the signal source.

In the case of a real-time application, it is preferable to use Cross-correlation due to the ease of implementation and the few computational resources to be used, as well as the processing speed, with the mention of a high sampling frequency.

The errors in the case of Spectrogram and Wavelet Transform are caused by the performance of *STFT*, approximations, and quantizations, the choice of parameters and their variation. The least satisfactory results are provided by the Spectrogram, and the Wavelet Transform is on the border between the three methods.

Errors for signal source localization are due to both the precision of the computational system, and the small time delay variations. The sampling frequency plays an important role in the case of a source localization system, influencing the time delay approximation; an error of microseconds or nanoseconds in the process of time delay estimation could represent an error of meters in signal source position estimation.

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